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Highly Cited Index for Technology (HCIT)
The Optimal Calculation Method to Determine the Effective Target Width for the Application of Fitts’ Law

SUMMARY In human-computer interaction, Fitts’ law has been applied in one-dimensional pointing task evaluation for some decades, and the usage of effective target width ($W_e$) in Fitts’ law has been accepted as an international standard in ISO standards 9241-9 [4]. However, the discussion on the concrete methods for calculating $W_e$ has not been developed comprehensively nor have the different methods of calculation been integrated. Therefore, this paper focuses on a detailed description and a comparison of the two main $W_e$ calculation methods. One method is mapping all the abscissa data in one unified relative coordinate system to perform the calculation (called CC method) and the other is dividing the data into two groups and mapping them in two separate coordinate systems (called SC method). We tested the accuracy of each method and compared both methods in a highly controlled experiment. The experiments’ results and data analysis show that the CC method is better than the SC method for human computer interface modeling. These results will be instrumental for future application of Fitts’ law.

key words: human-computer interaction, Fitts’ law, pointing task, effective target width

1. Introduction

Fitts’ law [3] is a famous model for one-dimensional pointing task evaluation in human computer interaction (HCI). In Fitts’ law experiment, subjects are usually required to point to two rectangle targets on a platform with a pen (or on a computer monitor with a mouse) reciprocally (see Fig. 1). Then the relationship between movement time ($MT$) and the index of difficulty ($ID_e$) are described in Eq. (1), a widely used form of Fitts’ law model [1], [6], [7].

$$MT = a + bID_e$$

$$ID_e = \log_2 \left( \frac{A}{W_e} + 1 \right)$$

$A$ is the amplitude between the centers of two rectangular targets, and $W_e$ is called the effective target width, which indicates the actual range of input hits around the target based on the performers’ actual behavior. The Fitts’ law model expressed by Eq. (1) has been accepted by ISO standards 9241-9 [4].

In Eq. (2), $W_e = 4.133SD$. $SD$ is the standard deviation of the hits distribution.

Although the Fitts’ law model defined by Eq. (1) has been used widely in HCI and advocated by many researchers [6], it is still not universally accepted [9]. One problem is that the calculation of $W_e$ has not been integrated.

Here we use Fig. 2 to describe the two calculation methods of $W_e$. As shown in Fig. 2(a), in a Fitts’ law experiment, the subjects’ input hits fall around the two rectangles. The two bell-like curves indicate the hits’ distribution near the left and right rectangles. Figure 2(b) indicates that some researchers use one unified coordinate system to calculate the average of the x-coordinates to get $SD$ and to calculate $W_e$, as mentioned in Douglas, Kirkpatrick and Mackenzie’s research [2]. We call this method the Combined-Coordinate Method (the CC method) in this paper. Other researchers use two sets of coordinate systems to calculate the average of the x-coordinates to get $SD$ and to calculate $W_e$ (see Fig. 2(c), as Isokoski and Raisamo have done in their study [5]. In this method the averages of the x-coordinates need to be calculated for the left and right coordinate systems respectively. We call this method the Separate-Coordinate Method (the SC method) in this paper.

However, at present, no research has been reported on the preferred method of $W_e$ calculation for the application of Fitts’ law. Moreover, no comparison has been reported although the usage of $W_e$ has been included in the ISO standard 9241-9 [4]. Therefore, in this paper we compare the two methods to see which one is better for calculating $W_e$. The results derived from this work will be of great help for the further application of Fitts’ law to the HCI field.

2. Testing Experiment: Testing the Hits’ Distribution for the SC Method

The SC method is much more complex than the CC method, but some researchers still support the SC method because they hold to the hypothesis that with bigger targets the users tend to click near the nearest edge of the rectangular target rather than near the middle of it. They therefore go on ar-
The subjects were required to perform the tapping task as fast and accurately as possible, as described in Fitts’ paradigm experiment[3]. During the task, except for the sound indicating a mistake had occurred, there was no other feedback to affect the subjects’ performance.

2.4 Results

Table 1 shows the $SD$, $ID_e$, and the corresponding amplitude and target width combinations in the Testing Experiment.

Table 1 The $SD$ and $ID_e$ with the CC method and the SC method in the Testing Experiment of the pointing task.

<table>
<thead>
<tr>
<th>Methods</th>
<th>Combinations</th>
<th>$SD$ (in pixels)</th>
<th>$ID_e$ (in pixels)</th>
</tr>
</thead>
<tbody>
<tr>
<td>the CC</td>
<td>$A=120, W=12$</td>
<td>3.44</td>
<td>3.24</td>
</tr>
<tr>
<td>Method</td>
<td>$A=120, W=36$</td>
<td>8.77</td>
<td>2.11</td>
</tr>
<tr>
<td></td>
<td>$A=120, W=72$</td>
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</tr>
<tr>
<td></td>
<td>$A=360, W=12$</td>
<td>3.29</td>
<td>4.78</td>
</tr>
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<td></td>
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</tr>
<tr>
<td></td>
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<td></td>
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<td>8.66</td>
<td>4.61</td>
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<tr>
<td></td>
<td>$A=840, W=72$</td>
<td>15.32</td>
<td>3.83</td>
</tr>
<tr>
<td>the SC</td>
<td>$A=120, W=12$</td>
<td>3.39</td>
<td>3.26</td>
</tr>
<tr>
<td>Method</td>
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<td>7.75</td>
<td>2.25</td>
</tr>
<tr>
<td></td>
<td>$A=120, W=72$</td>
<td>10.07</td>
<td>1.96</td>
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<td></td>
<td>$A=360, W=12$</td>
<td>3.27</td>
<td>4.79</td>
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<td></td>
<td>$A=840, W=72$</td>
<td>15.00</td>
<td>3.86</td>
</tr>
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</table>

The $SD$ and $ID_e$, with the CC method and the SC method in the Testing Experiment of the pointing task.

The experimental procedure was designed according to the ISO 9241-9 standard[4]. During the experiment, participants did reciprocal pointing with a mouse at a pair of vertical strip targets displayed on the screen. The width ($W$) of the targets and the center-to-center distances or amplitudes ($A$) between the two strips were set at $W=12, 36, 72$ pixels and $A=120, 360, 840$ pixels. The order of the nine width and distance combinations was randomized. The start position of the cursor was the center of the screen. Twelve trials were presented in each $W-A$ combination, with the first tap excluded in analysis. If the user tapped on the outside of the target, the task would not be abandoned and an auditory signal would be played.

The subjects were required to perform the tapping task as fast and accurately as possible, as described in Fitts’ paradigm experiment[3]. During the task, except for the sound indicating a mistake had occurred, there was no other feedback to affect the subjects’ performance.

2.5 Discussion

Table 1 shows that the values of $SD$ when using the SC method are less than when using the CC method, which in turn increases the values of $ID_e$. However the amount of change of $SD$ is uneven. For big target sizes, the SC method decreases the $SD$ more; for small target sizes, the SC method decreases the $SD$ slightly or does not decrease.

†In the Comparison Experiment, with each of the subjects’ taps there was an instant feedback signal appearing in the screen to remind the subjects to slow down or hurry up.
Fig. 3 The distribution of the input hits. (\(A = 120\) pixels, \(W = 36\) pixels)

Fig. 4 The distribution of the input hits. (\(A = 120\) pixels, \(W = 72\) pixels)

Fig. 5 The distribution of the input hits. (\(A = 840\) pixels, \(W = 36\) pixels)

Fig. 6 The distribution of the input hits. (\(A = 840\) pixels, \(W = 72\) pixels)

Isokoski and Raisamo can be demonstrated only by small amplitude (\(A = 120\)). When \(A\) was 120 pixels, for the left target, the distribution of dots leaned to the right slightly (Fig. 3 (a) and 4(a)), and for the right target, the distribution of dots leaned to the left slightly (Fig. 3 (b) and 4(b)). Never-
theless, with big amplitude \((A = 840)\), the off-center tendency is not clearly demonstrated with either bigger sizes \((W = 72\) pixels) or smaller sizes \((W = 36\) pixels): for the left target, the distribution of the dots did not lean obviously to the right of the center (Fig. 5 (a) and Fig. 6 (a)), meanwhile, the distribution of the dots around the right target did not lean obviously to the left of the center (Fig. 5 (b) and Fig. 6 (b)).

Through the Testing Experiment, we observed that the assumption of Isokoski and Raisamo was not applicable for all conditions. That meant we needed to do more work to compare the CC method and SC method. Therefore, we carried out a Comparison Experiment.

3. Comparison Experiment: Comparing the CC Method and the SC Method

Although the Testing Experiment has shown that Isokoski and Raisamo’s assumption was not completely supported, a clear comparison between the CC method and SC method could not be given only through the Testing Experiment. Therefore, we were intrigued to develop another experiment to concretely check which method of \(W_e\) calculation is better.

To analyze and compare the two methods of \(W_e\) calculation accurately, we developed an experiment that could produce a set of time measurements when participants kept their tapping within the given target widths to an almost ideal extent.

Since the results would be obtained from the ideal experimental situation, we expected to see a more precisely defined difference between the two methods.

3.1 Subject

The same subjects in the Testing Experiment participated in the Comparison Experiment.

3.2 Apparatus

The same apparatus in the Testing Experiment was applied in the Comparison Experiment, but the program was different because it was designed for different experimental purposes.

3.3 Design

In the Comparison Experiment, participants reciprocally pointed with a mouse on a pair of vertical strips which were at a fixed distance apart \(A\) of 400 pixels \(^1\). \(W\) (appointed target width) was set at 10, 14, 20, 28 and 40 pixels.

If the outside region of the target was tapped, the task would not be abandoned and an auditory signal would be played as a warning signal. The start position of the cursor for both parts was the center of the screen.

We used a target width enforcement method inspired by the verbal feedback method of Zhai and colleagues \([8],[10]\) to get the data when the subjects strictly complied with the required parameters of the program and pointed only within the target width. The purpose of this design is that by observing the ideal input hits distribution, we can see whether either of the methods is superior in modeling a pointing task.

During the experiment, if the participant took too much risk and produced a big \(SD\) and hence a big \(W_e\), the program would remind the performer to slow down via a real-time signal which appeared on the screen. In contrast, if \(W_e\) is very small, the program would remind the participant to hurry up. If the participant’s current endpoints dispersion corresponded to the ideal situation \((W = W_e\) within 7\% margin)\([10]\), no signal would appear and the participant was able to maintain his or her current pace. The judging thresholds for the different target sizes were shown in Table 2.

3.4 Procedure

We applied the following procedures for the CC method and SC method to calculate \(SD\) and control the program for the CC method and SC method.

For the CC method, the program calculated the \(SD\) based on a one coordinate system (see Fig. 2 (b)). It meant that the \(SD\) could be calculated by:

\[
SD = \frac{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}}{n - 1}
\]  

(3)

In Eq. (3), \(x_i\) was the \(i\)th of the participant’s selection point’s x-coordinates (They were mapped into one unit coordinate system). \(\bar{x}\) was the mean of x-coordinates. \(n\) was the number of the trials.

For the SC method, the situation was more complex. The program calculated \(SD\) based on two sets of coordinate systems (see Fig. 2 (c)). The concrete steps were as follows: first, to compute the averages of the left and right x-coordinates of the previous 14 trials (or less than this number before the 15th trial), secondly, to get the \(x_{i-x_{average}}\), \((i = 1, 2, \ldots, n, n \leq 14)\), here \(x_i\) was the \(i\)th hit’s x-coordinate, and \(x_{average}\) was the average of the values of \(x_i\), then there

\(^1\)In Fitts’ law studies, researchers agree that amplitude plays a much less important role in pointing tasks than target width \([6],[10]\). In the Comparison Experiment, when we focus on the problem of distribution, it is necessary to fix the less important variables so as to simplify the problem for the purpose of comparison. Furthermore, we wanted to observe a common experimental environment in pointing tasks. So we fixed the amplitude at 400 pixels, which is about half way between the smallest value (120 pixels) and the biggest value (840 pixels) of \(A\).
should be 14 numbers of $x_i-x_{\text{average}}$. (One point noticeable here was that for the left side hits and right side hits, the values of $x_{\text{average}}$ were different\footnote{For the left side hits, $x_i-x_{\text{average}}$ should be written as $x_{\text{left}}-x_{\text{average left}}$, and for the right side hits, $x_i-x_{\text{average}}$ should be written as $x_{\text{right}}-x_{\text{average right}}$.}, here we used $x_i-x_{\text{average}}$ only for the convenience of the following narration. The next step was to get the $SD$ of the 14 $(x_i-x_{\text{average}})$s, if $x'_i=x_i-x_{\text{average}}$, then

$$SD' = \sqrt{\frac{\sum_{i=1}^{n} (x'_i - \bar{x}')^2}{n-1}} \quad (4)$$

For both the CC method and the SC method, the procedure of measuring the running $W_e$ value was as follows: Before the participant performed the 15th trial in a $W$ condition, the program calculated the $SD$ of the end points based on all of the previous trials. From the 15th trial the program calculated the $SD$ of the end points based on the immediately preceding 14 trials. The experimental program stopped the current $W$ condition and began the next one once a block of 14 trials whose $W_e$ matches $W$ within a less than 7% margin was obtained. These 14 trials were used in later analysis. The program would have also aborted the current $W$ condition if the participant had performed 30 trials without reaching a 14 trial block that met the required balance between speed and accuracy. In the actual experiment none of the participants needed to use up the maximum 30 trials.

With either $W_e$ calculation method, the total amount of data for analysis was 700 (14 (trials) $\times$ 10 (subjects) $\times$ 5 (combinations of $A$ and $W$) = 700).

3.5 Results

After the experiment, we collected data and drew the Fitts’ law regression lines in Figs. 7 and 8.

In Fitts’ law, the relationship between movement time and target width is a logarithm relationship (Eq. (1) and Eq. (2)). Therefore, a logarithm relation curve between movement time and $W_e$ will be more helpful to compare the effect of the two calculation methods. Therefore, we also made the logarithmic regression lines between the $MT$ and $W_e$ based on the data of the experiment (Fig. 9 and Fig. 10).

3.6 Discussion

In Fig. 7, $R^2$ of the regression line of the CC method is near to 1 (0.989), which means that by using the CC method the regression of Fitts’ law is ideal and strong. The regression of Fitts’ law line in Fig. 8 is still big (0.909), but not as great as indicated by Fig. 7. This means that the SC method is not as precise as the CC method.

Fig. 9 shows that with the CC method, the logarithm relationship between movement time and effective target width is obvious and all five dots are restricted to the curve ($R^2 = 0.988$). However, in Fig. 10, the dots are scattered around the logarithm curve and are not confined tightly to the curve ($R^2 = 0.907$).

Since in the Comparison Experiment, the system gave an immediate response to the subject for each trial, the performance was under almost ideal control, therefore, the regression between $MT$ and $ID_e$ and the regression between $MT$ and $W_e$ was expected to be rather strong. From this point of view, the regression of the Fitts’ law line in Fig. 8 and the logarithmic regression in Fig. 10 (related to the SC method) are not strong enough.
4. General Discussions and Conclusions

The data from the uncontrolled Testing Experiment can help us to investigate the reason for the inadequacies in the SC method.

As explained previously, the values of $W_e$ calculated by the SC method decrease from those values calculated by the CC method, and the changing amount for different combinations of $A$ and $W$ are different (see Table 1). For big target sizes, the SC method decreases $SD$ and $W_e$ more; for small target sizes, the SC method decreases $SD$ and $W_e$ slightly. This irregular variation of $SD$ or $W_e$ obtained from using the SC method will result in a weaker regression between the mean time and $ID_e$ than the regression obtained by using the CC method. These results show that the use of the SC method produces irregular effects on different target sizes.

In the highly controlled Comparison Experiment, for the SC method, we used the two sets of coordinate systems to calculate $SD$, which means the requirements placed on the individual subject were less rigid than if we had used a one coordinate system. Nevertheless, when we analyze the data, we must mix all the subjects’ data together, and the $SD$ for all the dots will then be inflated. That is the reason why the effective target width obtained from the SC method is bigger than expected.

Based on the above analysis, it is logical to conclude that if we employed an effective coordinate system to calculate the effective target width, as much easier and more convenient than the SC method.

Another point worthy of note is that all the subjects in the two experiments were right-handed. Since for the left-handed person, the situation can simply be reversed, we can assume that the preferred hand will not affect the analytical results of this study.

In conclusion, we studied and compared two methods for calculating $W_e$. The results show that the CC method (Combined-Coordinate Method) is better than the SC method (Separate-Coordinate Method), i.e., it is better to map all the abscissa data into one integrated coordinate system to do the calculation, rather than to divide the data into two separate groups according to the corresponding target positions.

We believe that the data shown by this paper affords a detailed and reliable comparison of the two methods of $W_e$ calculation based on the information derived from the input hits with different target sizes. The Combined-Coordinate method recommended in this study will help researchers and developers determine more confidently and precisely the optimum effective target widths calculation method for pointing tasks.

References